Note to the Teacher





Please read this before you duplicate the Student Workbook.



This Smart Skies™ FlyBy Math™ Student Workbook contains activities for one Air Traffic Control problem. In particular, the Workbook includes worksheets for 6 different calculation methods your students can use to solve the problem:

---Count feet and seconds

--- Draw and stack blocks

---Plot points on two vertical lines

---Plot points on a grid

- ---Use the distance-rate-time formula
- ---Graph two linear equations

You will most likely want to **assign only 1 or 2 of the calculation methods**. So we recommend that you follow these steps before you duplicate the Workbook.



1. Select and Keep:

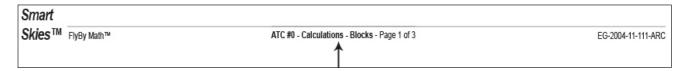
Choose the calculation method(s) you want your students to use.



2. Recycle the Rest:

Remove the calculation worksheets you do not wish to assign.

To find the calculation worksheets, look for the Calculations footers at the page bottoms.





Other activity worksheets follow the Calculation pages. Be sure you remove the Calculation worksheets ONLY.

For more information about the Smart Skies™ workbooks, please see the Smart Skies™ teacher materials available online at

Smart
Skies™ FlyBy Math™

http://smartskies.arc.nasa.gov

FlyBy Math™ Workbook Cover Letter EG-2004-11-111-ARC



FlyBy Math

Math & Science for Air Traffic Control



Problem #4

- Aircraft are on merging routes
- Aircraft are traveling at different speeds
- Aircraft start at different distances from where the routes meet

STUDENT WORKBOOK

Smart Skies™

Skies TM FlyBy Math™

READ THE PROBLEM

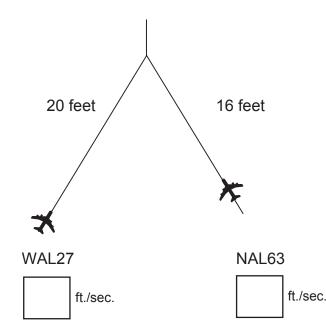




Begin Your Challenge: Will two planes, flying on merging jet routes, meet where the routes intersect?

Flight WAL27

- WAL27 is 20 feet (6.1 meters) from where the routes come together.
- The WAL27 speed is ½ foot per second (0.15 meters/second).
- Write the speed of WAL27 in the box below its picture.
- How far does WAL27 travel in 1 second?
- How far does WAL27 travel in 10 seconds?



Flight NAL63

- NAL63 is 16 feet (5.0 meters) from where the routes come together.
- The NAL63 speed is $^{1}/_{3}$ foot per second (0.10 meters/second).
- Write the speed of NAL63 in the box below its picture.
- How far does NAL63 travel in 1 second?
- How far does NAL63 travel in 10 seconds?

To meet your Challenge, you will:

- · Conduct an experiment.
- · Do some math calculations.
- · Analyze your results.

Then, you will use your results to answer these questions:

- · Will the planes meet at the point where the routes intersect?
- If not, how many feet apart will the planes be when the first plane reaches the point where the routes intersect?





SET UP and DO THE EXPERIMENT





Begin Your Task: Experiment to see what happens when the first plane reaches the point where the routes meet.

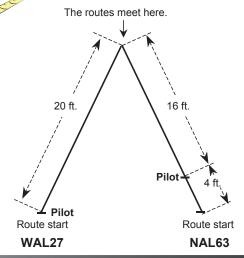
Flight WAL27

- Begin at the point where the routes meet.

 Use chalk or masking tape to mark off a line 20 feet long.
- At the jet route start, make a mark.

 Place a "Pilot" label next to the mark.

Lay out the jet routes.



Flight NAL63

- Begin at the point where the routes meet.

 Mark off a line 20 feet long.
 - The NAL63 pilot has a 4-foot headstart.
- Measure 4 feet from the route start and make a mark.

Place a "Pilot" label next to the mark.

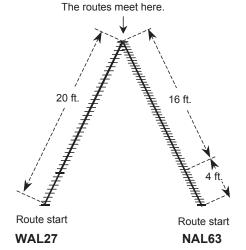


Flight WAL27

 $\binom{5}{5}$ Begin at the start of the jet route.

Place a mark (or piece of tape) every $\frac{1}{2}$ foot (6 inches) along the jet route all the way to the point where the routes meet.

Mark the speed control lines.



Flight NAL63

Begin at the start of the jet route.

Place a mark (or piece of tape) every $^{1}/_{3}$ foot (4 inches) along the jet route all the way to the point where the routes meet.



Investigator:
Investigator:

SET UP and DO THE EXPERIMENT





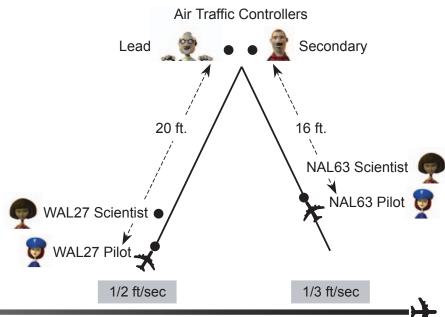
 $\binom{7}{7}$

Fill in this table with the starting conditions:

Flight Number	Speed (ft/sec)	Distance from the point where the routes meet (ft)
WAL27		
NAL63		

Circle your role on the ro

Circle your role on the route diagram on the right.





Do you where the	think the planes wil he routes come tog	ll meet at the p jether?	point	(10)	Why or why not?	
Ye	es	No				
Smart						

SET UP and DO THE EXPERIMENT



Circle your role and the steps below it.

Then, do the experiment 3 times using the steps for your role.





Steps	Lead Air Traffic Controller	Pilot	Secondary Air Traffic Controller	NASA Scientist
Take Your Position	Give the command: "Take your positions."	Stand at the Pilot mark on your jet route. Put one foot on each side of your route.	Go to your position.	Take your measuring tape to your jet route. Stand a few feet ahead of your pilot.
2 Get Ready to Start	Give the command: "Ready."	Practice stepping down your route.		
3 Start the Experiment	Start your stopwatch. Count seconds: "1", "2",	Take your first step on "1". On each count, take a step to the next mark.	Wait for the first pilot to reach the point where the routes meet.	Move ahead of the pilot.
4 Stop the Experiment	When you hear "Halt", stop counting seconds.	When you hear "Halt", stop where you are.	When the first pilot arrives, say "Halt". Record the seconds.	After you hear "Halt", measure and record the distance between pilots.

Record the data.



Record the Halt Time measured by the Secondary Controller.

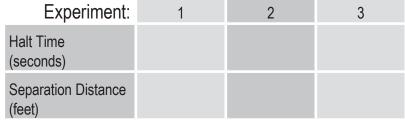


Record the Separation Distance measured by the NASA Scientist.



Use the results of your three experiments to choose the best answer to this question:







DO THE CALCULATIONS - Count Feet and Seconds





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Flight WAL27

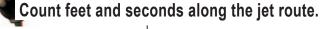
- Starts where its route begins (at 0 feet).
- In 1 second, it moves $\frac{1}{2}$ foot.
- In 2 seconds, it moves 1 foot from 0 feet to 1 foot.
- On the WAL27 jet route, find the 1-foot mark and trace "2 sec."
 - In the next 2 seconds (4 seconds total), WAL27 moves another foot from 1 foot to 2 feet.
- On the WAL27 jet route, find the 2-foot mark and trace "4 sec."
- Keep going on the WAL27 jet route, one foot at a time, until you reach the point where the routes meet.

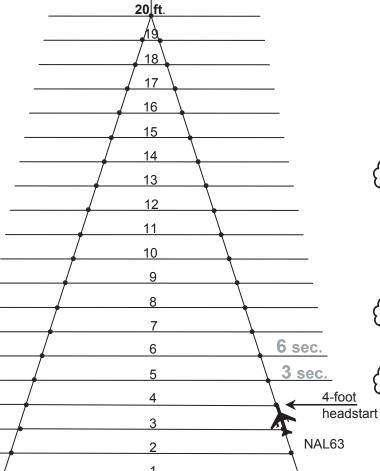
At each foot-mark, write the total number of seconds to reach that mark.

4 sec.

2 sec.

WAL27 T





Flight NAL63

- Starts 4 feet from the point where the route begins (a 4-foot headstart).
- In 1 second, it moves $\frac{1}{3}$ foot.
- In 3 seconds, it moves 1 foot from 4 feet to 5 feet.
- On the NAL63 jet route, find the 5-foot mark and trace "3 sec."
 - In the next 3 seconds (6 seconds total), NAL63 moves another foot from 5 feet to 6 feet.
- On the NAL63 jet route, find the 6-foot mark and trace "6 sec."
 - Keep going on the NAL63 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.



n feet

Investigator:	
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DO THE CALCULATIONS - Count Feet and Seconds



	Allswel tile q	ุนธิงแบทง.			
How many seconds did it take each plane to arrive at the point where the routes intersect?	WAL27	seconds	NAL63	seconds	
Did the planes meet at the point where the two routes inte	ersect? Y	ES	NO		
If No, which plane arrived first? WAL27	NAL63				
How many seconds did it take this plane to travel to the po	pint where the two	routes intersect?		seconds	
At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane to	raveled? How ma	feet ny feet was it from	the intersection?)	
If you think two planes will meet, what would you tell the ai	iir traffic controller	to do to avoid a co	ollision?		
You moved along each jet route, one foot at a time, to find Can you think of a faster way to find the number of second			plane to travel to	the point where	the routes meet.
Smart				End	of worksheet





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



Ise blocks to picture feet and seconds.



In this table, fill in the distance Flight WAL27 will travel in 10 seconds.

The speed of Flight WAL27 is $\frac{1}{2}$ foot per second.

Flight WAL27 takes	1 second	2 seconds	10 seconds
to travel	½ foot	1 foot	feet

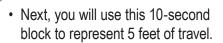


3.3 ft. 10 sec.

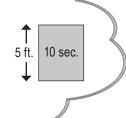
In this table, fill in the distance Flight NAL63 will travel in 10 seconds.

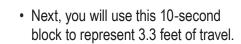
The speed of Flight NAL63 is $^{1}/_{3}$ foot per second.

Flight NAL63 takes	1 second	3 seconds	10 seconds
to travel	¹ / ₃ foot	1 foot	feet



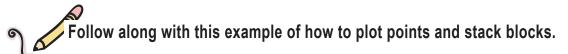
• The block height represents 5 feet, the distance Flight WAL27 travels in 10 seconds.





• The block height represents 3.3 feet, the approximate distance Flight NAL63 travels in 10 seconds.



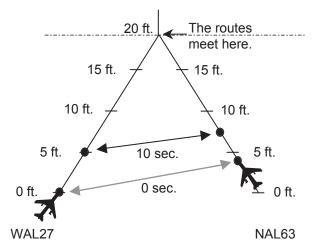


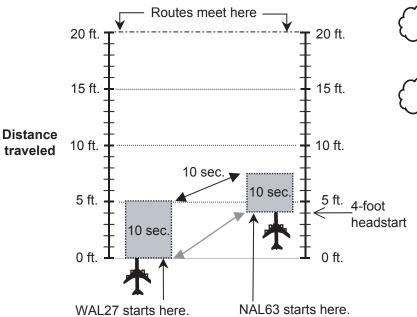
Flight WAL27

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.
- Circle the 5-foot point on the WAL27 jet route.

traveled

Trace the 10-second block for WAL27.





Flight NAL63

- Starts 4 feet from the point where the route begins (a 4-foot headstart).
- In 10 seconds, it moves approximately 3.3 feet.
- Now it is 7.3 feet along its route.
- Circle the 7.3-foot point on the NAL63 jet route.
- Trace the 10-second block for NAL63.

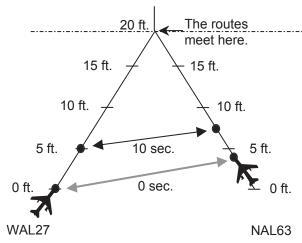


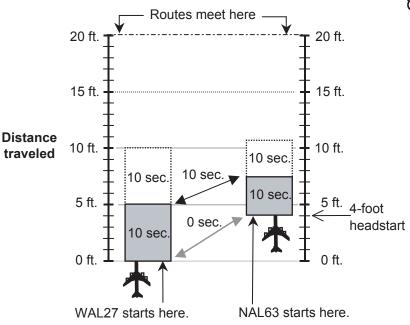


Now it's your turn to draw and connect.

Flight WAL27 and Flight NAL63

- On the route, draw a dot to show the position of WAL27 after 20 seconds.
- Trace the block on the graph below that shows the position of WAL27 after 20 seconds.
- On the route, draw a dot to show the position of NAL63 after 20 seconds.
- Trace the block on the graph that shows the position of NAL63 after 20 seconds.
- Connect your dots with a line marked "20 sec."
- Connect your blocks with a line marked "20 sec."





Now draw and connect at 30 seconds.

- Draw dots and blocks at 30 seconds.
- Connect dots and connect blocks at 30 seconds.

Keep going...

Keep going until the first plane reaches the point where the routes meet.

Be sure to connect your dots and connect your blocks.



Investigator:	
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Did the planes meet at the point where the two routes intersect? YES NO	
If No, which plane arrived first? WAL27 NAL63	O
How many seconds did it take this plane to travel to the point where the two routes intersect?	
At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)	
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?	

Smart Skies™ FlyBy Math™

DO THE CALCULATIONS - Plot Points on Lines





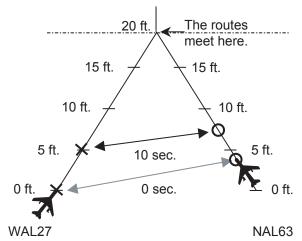
Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

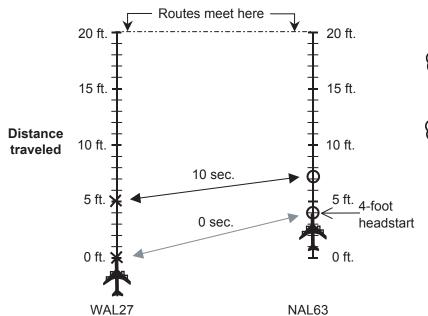
Follow along with this example of how to plot points.

Flight WAL27

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.
- Circle the **X** at the 5-foot point on the WAL27 jet route.
- Circle the **X** at the 5-foot point on the WAL27 line graph.

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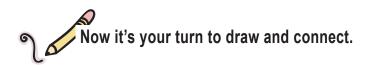


Flight NAL63

- Starts 4 feet from the point where its route begins (a 4-foot headstart).
- In 10 seconds, it moves approximately 3.3 feet.
- Now it is 7.3 feet along its route.
- Circle the **O** at the 7.3-foot point on the NAL63 jet route.
- Circle the **0** at the 7.3-foot point on the NAL63 line graph.

DO THE CALCULATIONS - Plot Points on Lines





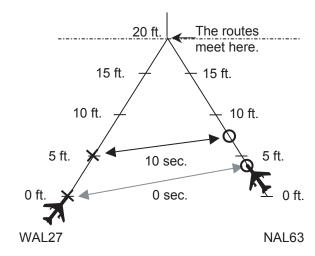
Flight WAL27 and Flight NAL63

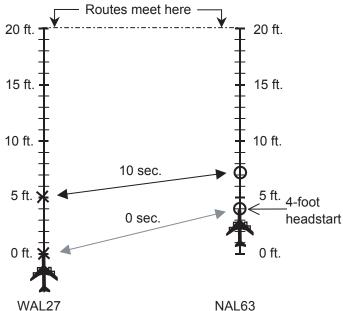
- On the route, draw an **X** to show the position of WAL27 after 20 seconds.
- On the line graph, draw an **X** to show the position of WAL27 after 20 seconds.
- On the route, draw an **0** to show the position of NAL63 after 20 seconds.
- On the line graph, draw an **0** to show the position of NAL63 after 20 seconds.
- On the routes, connect your X and O with a line marked "20 sec".

Distance

traveled

On the line graph, connect your X and **O** with a line marked "20 sec".





Now draw and connect at 30 seconds.

- On the routes, draw, connect, and label an **X** and an **O** at 30 seconds.
- On the graph, draw, connect, and label an **X** and an **O** at 30 seconds.

Keep going...

- Keep going until the first plane reaches the point where the routes meet.
 - Be sure to connect an X and an O on the route and on the graph.



DO THE CALCULATIONS - Plot Points on Lines



Answer the questions.	
Did the planes meet at the point where the two routes intersect? YES NO	
If No, which plane arrived first? WAL27 NAL63	
How many seconds did it take this plane to travel to the point where the two routes intersect?	
At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)	
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?	八

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DO THE CALCULATIONS - Plot Points on a Grid





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Follow along with this example of how to plot points.

20 ft.

10 sec.

0 sec.

15 ft.

10 ft.

5 ft.

0 ft.

WAL27

The routes meet here.

10 ft.

5 ft.

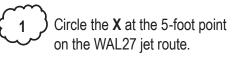
0 ft.

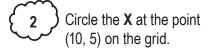
NAL63

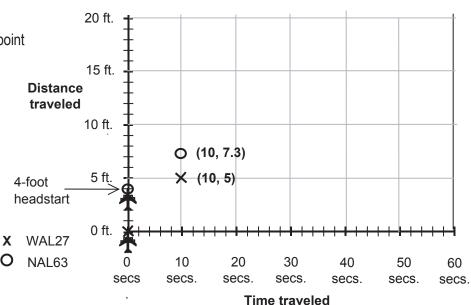
15 ft.

Flight WAL27

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.





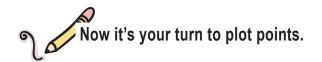


Flight NAL63

- Starts 4 feet from the point where the route begins (a 4-foot headstart).
- In 10 seconds, it moves approximately 3.3 feet.
- Now it is 7.3 feet along its route.
- Circle the **O** at the 7.3-foot point on the NAL63 jet route.
- Circle the **O** at the point (10, 7.3) on the grid.

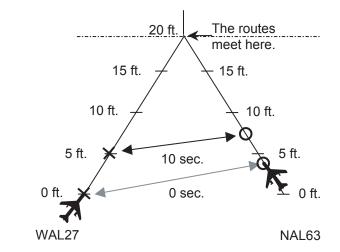
DO THE CALCULATIONS - Plot Points on a Grid

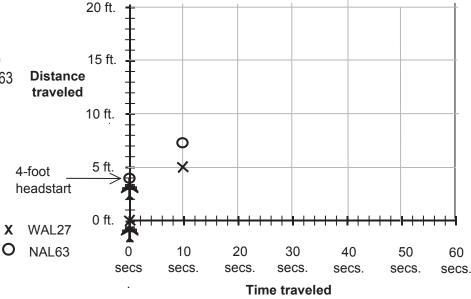




Flight WAL27 and Flight NAL63

- On the route, draw an **X** to show the position of WAL27 after 20 seconds.
- On the grid, draw an **X** to show the position of WAL27 after 20 seconds.
- On the route, draw an **0** to show the position of NAL63 after 20 seconds.
- On the grid, draw an **O** to show the position of NAL63 after 20 seconds.
- On the routes, connect your **X** and **O** with a line marked "20 sec".





Now draw and connect at 30 seconds.

- On the routes, draw, connect, and label an **X** and an **O** at 30 secs.
- On the grid, draw an **X** and an **O** at 30 seconds.

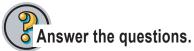
Keep going...

Keep going until the first plane reaches the point where the routes meet.

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DO THE CALCULATIONS - Plot Points on a Grid





Answer the questions.	
Did the planes meet at the point where the two routes intersect? YES NO	WAL27 ???
If No, which plane arrived first? WAL27 NAL63	
How many seconds did it take this plane to travel to the point where the two routes intersect?	
At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)	
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?	

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DO THE CALCULATIONS - Use a Formula





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



• The speed of Flight WAL27 is 0.5 feet per second.

\sim	
(1)	

Use multiplication to fill in this table.

In this many seconds	Flight '	WAL27 travels	this m	nany	feet			
1	0.5	feet/second	X	1	second	=	0.5	feet
2	0.5	feet/second	X	2	seconds	=	1.0	foot
3	0.5	feet/second	X	3	seconds	=	1.5	feet
4		feet/second	X		seconds	=		feet
5		feet/second	X		seconds	=		feet

(2)	How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds?

- The pattern in the table suggests this rule:
 "To find the distance traveled, multiply the speed by the time traveled."
- In math and science, we often say "rate" instead of "speed."
- So we can write the rule like this:

 $distance = rate \times time$

- This relationship is called the Distance-Rate-Time Formula.
- We often write it like this:

Distance-Rate-Time Formula						
d	=	r	•	t		

Use the formula to answer this question:

How many feet does Flight WAL27
travel in 20 seconds?

feet

- The speed of Flight NAL63 is $^{1}/_{3}$ foot per second.
- Use the formula to answer this question:

How many feet does Flight NAL63 travel in 20 seconds?

Smart
Skies Th

feet





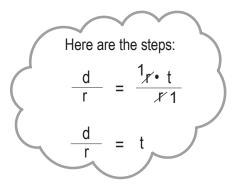
Distance-Rate-Time Formula

$$d = r \cdot t$$

If we divide both sides of the equation by r...

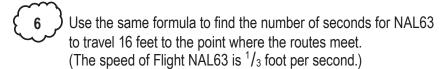
...then, we get a formula for time traveled.

$$t = \frac{d}{r}$$



$\binom{7}{5}$	Use this formula to find the number of seconds for WAL27
سن	to travel 20 feet to the point where the routes meet.

$$t = \frac{20 \text{ feet}}{0.5 \text{ feet per second}} = \boxed{\text{seconds}}$$

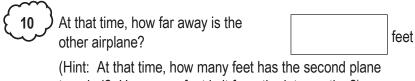


	seconds
--	---------

Will the planes meet at the point where the two routes intersect?

/ES [N	10

(g)	How many seconds will it take this plane	to travel to the	
	point where the two routes intersect?		seconds



traveled? How many feet is it from the intersection?)

he air



DO THE CALCULATIONS - Graph Linear Equations





Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.



Find an equation that describes the distance traveled by each plane.

Flight WAL27

We can use the Distance-Rate-Time formula

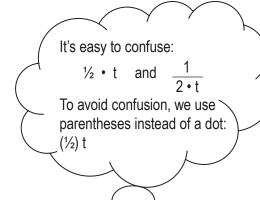
$$d = r \cdot t$$

to find d, the distance in feet WAL27 travels in t seconds.

• The WAL27 rate is $\frac{1}{2}$ foot per second.

So...

$$d = (\frac{1}{2}) t$$





Flight NAL63

We can use the Distance-Rate-Time formula

$$d = r \cdot t$$

to find d, the distance in feet NAL63 travels in t seconds.

- The NAL63 rate is ¹/₃ foot per second.
- NAL63 has a 4-foot headstart. (When you start your stopwatch at t=0, NAL63 has already traveled 4 feet.) So...

$$d = (1/3) t + 4$$

DO THE CALCULATIONS - Graph Linear Equations



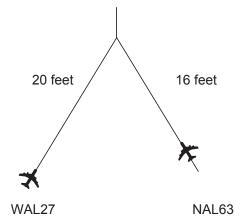


Fill in the table for WAL27.

$$d = (\frac{1}{2})t$$

t seconds	d feet
0	
10	
20	
30	
40	

Fill in each table.

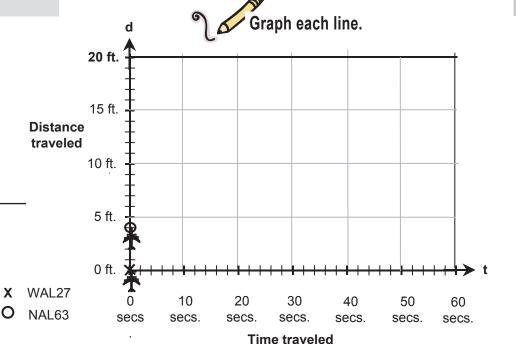




Fill in the table for NAL63.

$$d = (1/3)t + 4$$

t seconds	d feet
0	
10	
20	
30	
40	





Use an **O** to graph each point in the NAL63 table.

Use a dotted line • • • • • • • • to connect the points.

Use an X to graph each

point in the WAL27 table.

Use a solid line -

to connect the points.

DO THE CALCULATIONS - Graph Linear Equations



Answer the questions.

How many second the routes inters		ane to arrive at the point	t where	(10)	If you think two planes will meet, what would you tell air traffic controller to do to avoid a collision?	the
Will the planes of YES	meet at the point where	e the two routes intersec	ot?	(11)	Write the number that is the slope of the solid line representing WAL27.	
7 If NO, which pla	ne will arrive first?	NAL63		(12)	Write the number that is the slope of the dotted line representing NAL63.	
	onds will it take this pla nt where the two route		secs	(13)	What information does the slope of the line tell you a each plane?	ıbout
(Hint: At that tir	w far away is the other me, how many feet has is it from the intersection	the second plane travel	feet			

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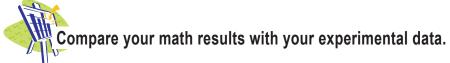
Investigator:

ANALYZE YOUR RESULTS





Your Task: Analyze and explain your results. Then apply what you learned to another problem.



		•					
$\binom{1}{1}$	Use your experimental data a	and math results	to fill in this table:	5	Use your experimental data	and your math res	sults to fill in this
		Experiment	Mathematics			Experiment	Mathematics
	Did the planes meet where the routes meet? (Yes or No)				Separation distance (feet) where the routes meet.		
(2)	Do your experimental and yo		natch?	6	Do your experimental and y		natch?
(3)	If No, why do you think they	don't match?		(7)	If No, why do you think they	don't match?	
4	Which is correct? Why?			(8)	Which is correct? Why?		

ANALYZE YOUR RESULTS





Compare the speeds and compare the distances.

9	Are the planes' speeds	the same or different?	Same	Diffe	rent
$\binom{10}{10}$	Fill in this table.				
		Which plane is ahead?	Separation Distance (feet)		
	At the start:	Flight	headstart = 4 feet		
	At the intersection:	Flight	Separation =	feet	
		2			



Consider the general problem.

Two planes are flying at different speeds on two different routes. The planes start at different distances from the point where the routes meet.



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Do you have enough information to predict whether the planes will meet at the point where the routes meet?

Yes

No

If NO, what other information do you need?

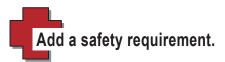


EXTENSION





Your Task: Will the two planes violate the separation standard where the routes meet?



For safety reasons, the planes must be separated by at least 5 feet at the point where the routes meet.



	Based upon your calculation, what is the difference in the planes' final positions? (That is, what is the planes' separation distance where the routes meet?)
2	Does this distance satisfy the separation requirement? Yes No
$\binom{3}{3}$	If NO, what would you tell the air traffic controller to do to meet the separation requirement?



feet

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